

Exercise 2

1 Quorum Systems using Trees

Consider a complete k -ary tree with n nodes.

- a) The set of all paths from leaf nodes to the root is a quorum system, where all quorums have uniform size $\lceil \log_k n \rceil$. What are its load and its resilience?
- b) Design a quorum system based on a tree with better resilience, but larger quorums.

2 Byzantine Quorum Systems

Quorum systems have been defined in the course for load balancing and for tolerating faulty servers. The possible failures were limited to crashes.

Consider now a *Byzantine quorum system*, i.e., one that tolerates arbitrary (Byzantine) failures, where faulty servers may give wrong answers. We want to solve the mutual exclusion problem. As with crash failures, the idea is that a server obtains permission from a quorum of nodes.

- a) Let $\mathcal{P} = \{P_1, \dots, P_n\}$ be a set of servers. Define a Byzantine quorum system on \mathcal{P} that tolerates t failures.
Hint: Make sure that some non-faulty server is in the intersection of every two quorums. Note that faulty servers may give wrong answers, but safety of the mutual exclusion must still be ensured.
- b) Based on the 2-dimensional Grid quorum system, describe a Byzantine quorum system that tolerates t faulty servers.